Chapter 4

1

Motion in two dimensions

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Lecture 01

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- Position, Velocity and Acceleration
- Kinematic Equations

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Position and Displacement

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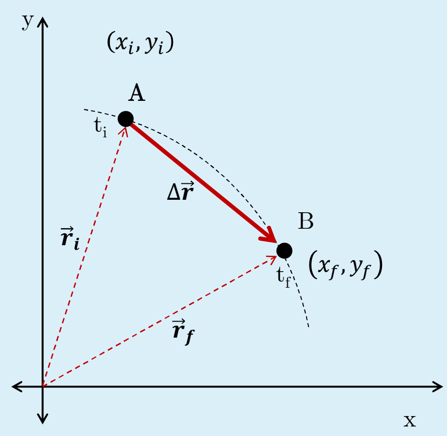
The position of an object is described by its position vector, $\vec{r}$, with respect to a chosen reference point (the origin).

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

The displacement of the object is the *change in its position.*

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{r} = (x_f - x_i)\hat{\imath} + (y_f - y_i)\hat{\jmath}$$



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Velocity and Acceleration

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Average Velocity:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous Velocity:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Average Acceleration:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

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Kinematic Equations for 2-D Motion

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- These equations will be similar to those of one-dimensional kinematics.
- Motion in two dimensions can be modeled as two independent motions in each of the two perpendicular directions associated with the x and y axes.
  - Any influence in the y direction does not affect the motion in the x direction.

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Kinematic Equations

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Since acceleration is constant, we can also find an expression for the velocity as a function of time:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

The position vector can also be expressed as a function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

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Kinematic Equations: 2-D

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Equations	Missing
$\vec{v}_f = \vec{v}_i + \vec{a}t$	$\Delta\vec{r}$: displacement (m)
$\Delta\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	$\vec{v}_f$: final velocity (m/s)
$\Delta\vec{r} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$	$\vec{v}_i$: initial velocity (m/s)
$\Delta\vec{r} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)t$	$\vec{a}$: acceleration (m/s^2)

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Kinematic Equations: Horizontal components

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Equations	Missing
$v_{fx} = v_{ix} + a_x t$	Δx : displacement (m)
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$	t : time (s)
$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$	v_{fx} : final velocity (m/s)
$\Delta x = v_{fx}t - \frac{1}{2}a_x t^2$	v_{ix} : initial velocity (m/s)
$\Delta x = \frac{1}{2}(v_{ix} + v_{fx})t$	a_x: acceleration (m/s^2)

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Kinematic Equations: Vertical components

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Equations	Missing
$v_{fy} = v_{iy} + a_y t$	Δy : displacement (m)
$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$	t : time (s)
$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$	v_{fy} : final velocity (m/s)
$\Delta y = v_{fy} t - \frac{1}{2} a_y t^2$	v_{iy} : initial velocity (m/s)
$\Delta y = \frac{1}{2}(v_{iy} + v_{fy})t$	a_y: acceleration (m/s^2)

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Average velocity

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A motorist drives south at $20\,m/s$ for $3min.$, then turns west and travels at $25\,m/s$ for $2min.$, and finally travels northwest at $30\,m/s$ for $1min.$. For this $6min.$ trip, find:

- the total vector displacement,
- the average speed
- the average velocity.

Average acceleration

Saturday, 30 January, 2021 12:22

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A car is traveling east at (60 km/h), it rounds a curve and after (5 s), it is traveling north at the same speed. Find the average acceleration of the car.

Position, velocity and acceleration

Saturday, 30 January, 2021 12:23

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$

(a)At $t = 15\text{s}$, what is the rabbit's position vector $\vec{r}$, in unit vector notation and in magnitude-angle notation?

(b) find the velocity $\vec{v}$ at time $t = 15s$.

(c) find the acceleration $\vec{a}$ at time $t = 15$ s.

SOLUTION

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66m$$

And

$$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57m,$$

So:

$$\vec{r} = (66m)\hat{\imath} - (57m)\hat{\jmath},$$

To get the magnitude and angle of $\vec{r}$, we use:

$$r = \sqrt{x^2 + y^2} = \sqrt{(66m)^2 + (-57m)^2}$$

$$= 87m,$$

And

$$\theta = tan^{-1}\frac{y}{x} = tan^{-1}\left(\frac{-57m}{66m}\right) = -41°.$$

Although θ=139° has the same tangent as $-41°$, the components of position vector $\vec{r}$ indicate that the desired angle is $139° - 180° = -41°$.

SOLUTION

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$

$$= -0.62t + 7.2.$$

At $t = 15$ s, this give $v_x = -2.1\text{m/s}$.. Similarly, applying the v_y part, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$

$$= 0.44t - 9.1.$$

At $t = 15s$, this gives

$$v_y = -2.5m/s.$$

$$\vec{v} = (-2.1m/s)\hat{\imath} + (-2.5m/s)\hat{\jmath},$$

To get the magnitude and angle

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1m/s)^2 + (-2.5m/s)^2} = 3.3m/s$$

$$\theta = tan^{-1}\frac{v_y}{v_x} = tan^{-1}\left(\frac{-2.5m/s}{-2.1m/s}\right)$$

$$= tan^{-1} 1.19 = -130°.$$

SOLUTION

the x component of $\vec{a}$ to be

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62m/s^2.$$

the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44m/s^2.$$

We see that the acceleration does not vary with time

$$\vec{a} = (-0.62m/s^2)\hat{\imath} + (0.44m/s^2)\hat{\jmath},$$

For the magnitude we have

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62m/s^2)^2 + (0.44m/s^2)^2} = 0.76m/s^2.$$

For the angle we have

$$\theta = tan^{-1}\frac{a_y}{a_x} = tan^{-1}\left(\frac{0.44m/s^2}{-0.62m/s^2}\right) = -35°.$$

However, this angle, which is the one displayed on a calculator, indicates that $\vec{a}$ is directed to the right and downward.Yet, we know from the components that $\vec{a}$ must be directed to the left and upward. To find the other angle that has the same tangent as -35° but is not displayed on a calculator, we add 180°:

$$-35° + 180° = 145°.$$

Uniform acceleration

Saturday, 30 January, 2021 12:24

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A particle moves in the xy plane, starting from the origin at $t = 0$ with an initial velocity having an x component of $20\, m/s$ and a y component of $-15\, m/s$. The particle experiences an acceleration in the x direction, given by $a_x = 4\, m/s^2$.

(A) Determine the total velocity vector at any time.

(B) Calculate the velocity and speed of the particle at $t = 5$ s and the angle the velocity vector makes with the x axis.

(C) Determine the x and y coordinates of the particle at any time (t) and its position vector at this time.

SOLUTION

We set $v_{xi} = 20m/s$, $v_{yi} = -15m/s$, $a_x = 4.0m/s^2$, and $a_y = 0$.

the velocity vector:

$$\vec{v}_f = \vec{v}_i + \vec{a}_t = (v_{xi} + a_x t)\hat{\imath} + (v_{yi} + a_y t)\hat{\jmath}$$

Substitute numerical values with the velocity in (m/s) and the time in seconds:

$$\vec{v}_f = [20 + (4.0)t]\hat{\imath} + [-15 + (0)t]\hat{\jmath} = [(20 + 4.0t)\hat{\imath} - 15\hat{\jmath}]$$

Notice that the x component of velocity increases in time while the y component remains constant;

SOLUTION

At $t = 5.0s$:

$$\vec{v}_f = [(20 + 4.0(5.0))\hat{\imath} - 15\hat{\jmath}] = (40\hat{\imath} - 15\hat{\jmath})m/s$$

Determine the angle θ that $\vec{v}_f$ makes with the x axis at $t = 5.0s$:

$$\theta = tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = tan^{-1}\left(\frac{-15m/s}{40m/s}\right) = -21°$$

Evaluate the speed of the particle as the magnitude of $\vec{v}_f$:

$$v_f = \left|\vec{v}_f\right| = \sqrt{v_{xf}^2 + _{yf}^2} = \sqrt{(40)^2 + (-15)^2 m/s} = 43m/s$$

The negative sign for the angle θ indicates that the velocity vector is directed at an angle of 21° below the positive x axis.

SOLUTION

With $x_i = y_i = 0$ at $t = 0$

Express the position vector of the particle at any time t:

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20t + 2.0t^2$$

$$y_f = v_{yi}t = -15t$$

$$\Rightarrow \vec{r}_f = x_f\hat{\imath} + y_f\hat{\jmath} = (20t + 2.0t^2)\hat{\imath} - (15t)\hat{\jmath}$$